1. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(b) Find the speed of B immediately after hitting the wall.

(2)

The first collision between A and B occurred at a distance 4a from the wall. The balls collide again T seconds after the first collision.

(c) Show that $T = \dfrac{112a}{15u}$.

(6)
(Total 15 marks)

2. Particles A, B and C of masses $4m$, $3m$ and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u m s^{-1} and v m s^{-1} respectively, and collide directly.

As a result of the collision, A is brought to rest and B rebounds with speed kv m s^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$.

(6)

(b) Find the value of k.

(2)

Immediately after the collision between A and B, particle C is projected with speed $2v$ m s^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B.

(4)
(Total 12 marks)

3. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$.

(5)

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$.

(4)

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall.

(4)

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall.

(4)
(Total 17 marks)

4. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(a) (i) Show that the speed of P immediately after the collision is $\frac{u}{6}(5e - 1)$.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

(6)

Three small spheres A, B and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(b) Show that, after B and C have collided, there is a collision between B and A.

(3)

(c) Determine whether, after B and A have collided, there is a further collision between B and C.

(4)
(Total 13 marks)

5. A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Find, in terms of v only, the speed of P before the collision.

(3)

(b) Find the value of k.

(3)

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(c) the coefficient of restitution between Q and R is $\frac{3}{4}$,

(4)

(d) there will be a further collision between P and Q.

(2)
(Total 12 marks)

6. Two particles A and B move on a smooth horizontal table. The mass of A is m, and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.

(a) Find expressions for the speed of A and the speed of B immediately after the collision.

(7)

In the subsequent motion, B strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{4}{5}$. Given that there is a second collision between A and B,

(b) show that $\frac{1}{4} < e < \frac{9}{16}$.

(5)

Given that $e = \frac{1}{2}$,

(c) find the total kinetic energy lost in the first collision between A and B.

(3)
(Total 15 marks)

7. A particle A of mass $2m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle B of mass m moving with speed $2u$ in the opposite direction to A. Immediately after the collision the speed of B is $\frac{8}{3}u$ and the direction of motion of B is reversed.

(a) Calculate the coefficient of restitution between A and B.

(6)

(b) Show that the kinetic energy lost in the collision is $7mu^2$.

(3)

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of B. The magnitude of the impulse of the wall on B is $\frac{14}{3}mu$.

(c) Calculate the coefficient of restitution between B and the wall.

(4)
(Total 13 marks)

8. Two small spheres A and B have mass $3m$ and $2m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2u$, when they collide directly. As a result of the collision, the direction of motion of B is reversed and its speed is unchanged.

(a) Find the coefficient of restitution between the spheres.

(7)

Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.

(b) Show that, after B collides with C, there will be no further collisions between the spheres.

(7)
(Total 14 marks)

9. Two small smooth spheres, P and Q, of equal radius, have masses $2m$ and $3m$ respectively. The sphere P is moving with speed $5u$ on a smooth horizontal table when it collides directly with Q, which is at rest on the table. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $2(1 + e)u$.

(5)

After the collision, Q hits a smooth vertical wall which is at the edge of the table and perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f, $0 < f \leq 1$.

(b) Show that, when $e = 0.4$, there is a second collision between P and Q.

(3)

Given that $e = 0.8$ and there is a second collision between P and Q,

(c) find the range of possible values of f.

(3)
(Total 11 marks)

10. A uniform sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another uniform sphere B of mass $2m$ which is at rest on the table. The spheres are of equal radius and the coefficient of restitution between them is e. The direction of motion of A is unchanged by the collision.

(a) Find the speeds of A and B immediately after the collision.

(7)

(b) Find the range of possible values of e.

(2)

After being struck by A, the sphere B collides directly with another sphere C, of mass $4m$ and of the same size as B. The sphere C is at rest on the table immediately before being struck by B. The coefficient of restitution between B and C is also e.

(c) Show that, after B has struck C, there will be a further collision between A and B.

(6)
(Total 15 marks)

11. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

(a) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively.

(7)

After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

(b) Find the value of e.

(7)

(c) Explain why there must be a third collision between P and Q.

(1)
(Total 15 marks)

1. (a) (i)

Con. of Mom: $3mu - mu = 3mv + mw$

$2u = 3v + w$ (1) M1# A1

N.L.R: $\frac{1}{2}(u+u) = w - v$ M1# A1

$u = w - v$ (2)

(1) – (2) $u = 4v$ DM1#

$v = \frac{1}{4}u$ A1

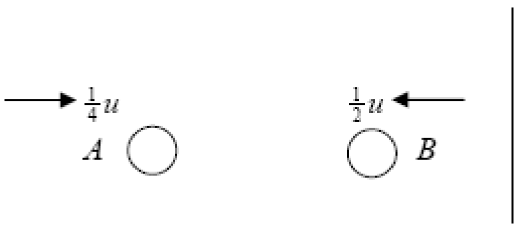
(ii) In (2) $u = w - \frac{1}{4}u$

$w = \frac{5}{4}u$ A1 7

(b) B to wall: N.L.R: $\frac{5}{4}u \times \frac{2}{5} = V$ M1

$V = \frac{1}{2}u$ A1ft 2

(c)



B to wall: $\text{time} = 4a \div \frac{5}{4}u = \frac{16a}{5u}$ B1ft

Dist. Travelled by $A = \frac{1}{4}u \times \frac{16a}{5u} = \frac{4}{5}a$ B1ft

In t secs, A travels $\frac{1}{4}ut$, B travels $\frac{1}{2}ut$

Collide when speed of approach $= \frac{1}{2}ut + \frac{1}{4}ut$, distance to cover = M1$

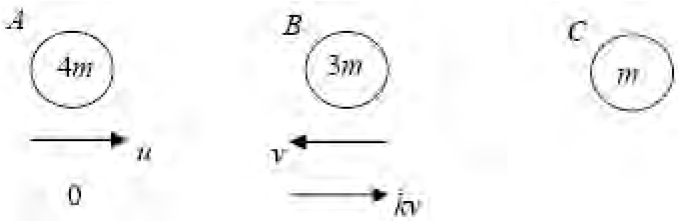
$4a - \frac{4}{5}a$

$\therefore\ t = \frac{4a - \frac{4}{5}a}{\frac{3}{4}u} = \frac{16a}{5} \times \frac{4}{3u} = \frac{64a}{15u}$ DM1$ A1

Total time $= \frac{16a}{5u} + \frac{64a}{15u} = \frac{112a}{15u}$ * A1 6

[15]

2. (a)



Conservation of momentum: $4mu - 3mv = 3mkv$	M1A1	
Impact law: $kv = \frac{3}{4}(u+v)$	M1A1	
Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u+v)$	DM1	
$u = 3v$ (Answer given)	A1	6

(b) $kv = \frac{3}{4}(3v+v), k = 3$ M1,A1 2

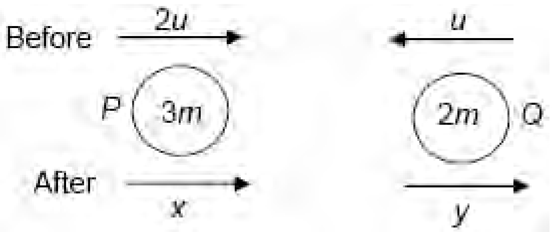
(c) Impact law: $(kv + 2v)\,e = v_C - v_B$ $(5ve = v_C - v_B)$ B1

Conservation of momentum : $3\times kv - 1 \times 2v = 3v_B + v_C$ $(7v = 3v_B + v_c)$ B1

Eliminate $v_C : v_B = \frac{v}{4}(7-5e) > 0$ hence no further collision with *A*. M1 A1 4

[12]

3. (a)



Correct use of NEL	M1 *	
$y - x = e(2u + u)$ o.e.	A1	
CLM (→): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ $(\Rightarrow 4u = 3x + 2y)$	B1	
Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, $(u(9e + 4) = 5y)$	d * M1	
Hence, speed of Q = $\underline{\tfrac{1}{5}(9e+4)u}$ **AG**	A1 cso	5

(b) $x = y - 3eu = \frac{1}{5}(9e+4)u - 3eu$ M1 *

Hence, speed $\text{P} = \frac{1}{5}(4-6e)u = \frac{2u}{5}(2-3e)$ o.e. A1

$x = \frac{1}{2}u = \frac{2u}{5}(2-3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e d * M1

gives, $e = \frac{3}{12} \Rightarrow \underline{e = \frac{1}{4}}$ **AG** A1

Or

Using NEL correctly with given speeds of P and Q M1 *

$3eu = \frac{1}{5}(9e+4)u - \frac{1}{2}u$ A1

$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e d * M1

$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$. A1 4

(c) Time taken by Q from A to the wall = $\frac{d}{\underline{y}} = \left\{\frac{4d}{5u}\right\}$ M1⁺

Distance moved by P in this time = $\frac{u}{2} \times \frac{d}{y} \left(= \frac{u}{2}\left(\frac{4d}{5u}\right) = \frac{2}{5}d\right)$ A1

Distance of P from wall = $d - \times\left(\frac{d}{y}\right); = d - \frac{2}{5}d = \underline{\frac{3}{5}d}$ **AG** d⁺M1;

A1 cso

or

Ratio speed P:speed Q = x:y = $\frac{1}{2}u : \frac{1}{5}(\frac{9}{4}+4)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$ M1⁺

So if Q moves a distance d, P will move a distance $\frac{2}{5}d$ A1

Distance of P from wall = $d - \frac{2}{5}d; = \underline{\frac{3}{5}d}$ **AG** cso d⁺M1;A1 4

(d) After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y B1ft

Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$ M1

gives, $2(\frac{3d}{5} - x) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \frac{3d}{5} \Rightarrow \underline{x = \frac{1}{5}d}$ A1 cao

or

After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ B1 ft
from their y

Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5}; = \frac{d}{5}$ their $\frac{1}{2+1}$ M1; A1 4

2nd or

After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y B1ft

Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$

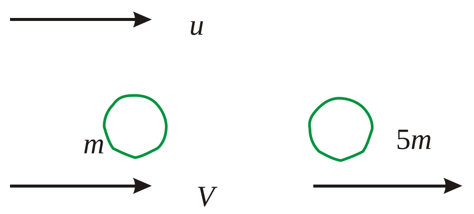
Time from wall to 2nd collision = $\dfrac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y B1ft

Distance of B from

wall = (their speed)x(their time) = $\dfrac{u}{4} \times \dfrac{4d}{5u}; = \underline{\frac{1}{5}d}$ M1; A1 4

[17]

4.



(a) CLM: $mv + 5mw = mu$ B1

NLI: $w - v = eu$ B1

Solve v: $v = \dfrac{1}{6}(1 - 5e)u$, so speed = $\dfrac{1}{6}(5e - 1)u$ M1*A1

(NB – answer given on paper)

Solve w: $w = \dfrac{1}{6}(1 + e)u$ M1*A1 6

* The M's are dependent on having equations (not necessarily correct) for CLM and NLI

(b) After B hits C, velocity of B = "v" = $\dfrac{1}{6}(1 - 5.\dfrac{4}{3})u = -½u$ M1A1

velocity < 0 $\Rightarrow$ change of direction $\Rightarrow$ B hits A A1CSO 3

(c) velocity of C after = $\dfrac{3}{10}u$ B1

When B hits A, "u" = ½u, so velocity of B after = –½(–½u) = $\dfrac{1}{4}u$ B1

Travelling in the same direction but $\dfrac{1}{4} < \dfrac{3}{10} \Rightarrow$ <u>no second collision</u> M1A1cso 4

B1 Conservation of momentum – signs consistent with their diagram/between the two equations

B1 Impact equation

M1 Attempt to eliminate w

A1 correct expression for v. Q asks for speed so final answer must be verified positive with reference to e > 1/5.

Answer given so watch out for fudges.

M1 Attempt to eliminate v

A1 correct expression for w

M1 Substitute for e in speed or velocity of P to obtain v in terms of u. Alternatively, can obtain v in terms of w.

A1 (+/–)u/2 $\left(v = -\dfrac{5w}{3}\right)$

A1 CSO <u>Justify direction</u> (and correct conclusion)

B1 speed of C = value of w = $(\pm)\dfrac{3u}{10}$ (Must be referred to in (c) to score the B1.)

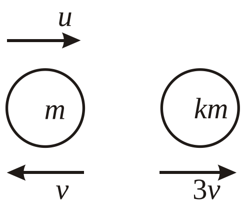
B1 speed of B after second collision $(\pm)\dfrac{1}{4}u$ or $(\pm)\dfrac{5}{6}w$

M1 Comparing their speed of B after 2nd collision with their speed of C after first collision.

A1 CSO. Correct conclusion.

[13]

5. (a)



NEL $3v - (-V) = eu$ M1A1

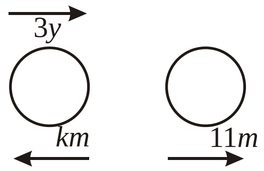
$u = 8v$ A1 3

(b) LM $8mv = -mv + 3kmv$ ft their u M1A1ft

$(m \times (u) = -mv + 3kmv)$

$k = 3$ A1 3

(c)



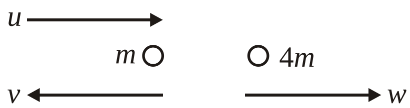
LM $9mv = -3my + 11my$	ft their k	M1A1ft	
NEL $2y = e \times 3v$		M1	
$y = \frac{9}{8}v \Rightarrow e = \frac{3}{4}$ *	cso	A1	4

(d) $y = \frac{9}{8}v > v \Rightarrow$ further collision between P and Q — M1A1 — 2

A1 is cso – watch out for incorrect statements re. velocity

[12]

6. (a)



$mu = 4mw - mv$	M1 A1	
$eu = w + v$	M1 A1	
$\Rightarrow w = (\frac{1+e}{5})u,\ v = (\frac{4e-1}{5})u$	Indep M1 A1 A1	7

(b)

$w' = (\frac{4+4e}{25})u$	B1 **f.t.**	
Second collision $\Rightarrow w' > v$		
$\Rightarrow \frac{4+4e}{25} > \frac{4e-1}{5}$	M1	
Also $v > 0 \Rightarrow e > 1/4$ Hence result	B1	5

(c)

KE lost = $\frac{1}{2}mu^2 - [\frac{1}{2}4m\{(u/5)(1+e)\}^2 + \frac{1}{2}m\{(u/5)(4e-1)\}^2]$	M1A1f.t.	
$= \frac{3}{10}mu^2$	A1 cao	3

[15]

7. (a)

LM $6mu - 2mu = 2mx + \frac{8}{3}mu$ M1 A1

$\left(x = \frac{2}{3}u\right)$

NEL $\frac{8}{3}u - x = 5ue$ M1 A1

Solving to $e = \frac{2}{5}$ M1 A1 6

(b) Initial K.E. = $\frac{1}{2} \times 2m(3u)^2 + \frac{1}{2} \times m(2u)^2 = 11mu^2$

Final K.E. = $\frac{1}{2} \times 2m\left(\frac{2}{3}u\right)^2 + \frac{1}{2} \times m\left(\frac{8}{3}u\right)^2 = 4mu^2$ both M1

Change in K.E. = $7mu^2$ M1 Subtracting and simplifying to kmu^2 M1 A1 A1cso 3

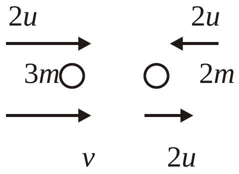
(c) $m\left(\frac{8}{3}u + v\right) = \frac{14}{3}mu$ M1 A1

($v = 2u$)

$e = \frac{2}{8/3} = \frac{3}{4}$ M1 A1 4

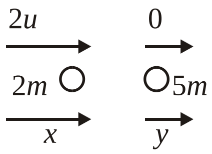
[13]

8. (a)



CLM: $6mu - 4mu = 3mv + 4mu$	M1 A1	
$\Rightarrow v = -\frac{2}{3}u$	A1	
NLI: $2u - v = e.4u$	M1 A1	
$\Rightarrow 4eu = \frac{8}{3}u \Rightarrow e = \frac{2}{3}$.	M1 A1	7

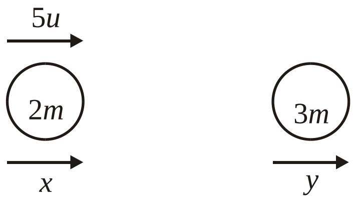
(b)



$5my + 2mx = 4mu$	M1 A1	
$y - x = \frac{3}{5}.2u = \frac{6}{5}u$	A1	
Solve: $x = -\frac{2}{7}u$	M1 A1	
$\frac{2}{7}u < \frac{2}{3}u$ so B does not overtake A	M1	
So no more collisions	A1cso	7

[14]

9.

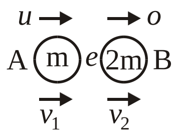


(a)	LM $10mu = 2mx + 3my$	M1 A1	
	NEL $y - x = 5eu$	B1	
	Solving to $y = 2(1 + e)u$ (*)	cso M1 A1	5
(b)	$x = 2u - 3eu$ finding x, with or without $e = 0.4$	M1	
	$x = 0.8u$	A1	
	$x > 0 \Rightarrow P$ moves towards wall and Q rebounds from wall $\Rightarrow$ second collision — ft any positive x	A1 ft	3

(c)	$x = -0.4u$		B1	
	Speed of Q on rebound is $3.6fu$			
	For second collision $3.6fu > 0.4u$		M1	
	$f > \frac{1}{9}$	ignore $f \le 1$	A1	3

[11]

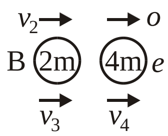
10. (a)



$mu = mv_1 + 2mv_2$	M1 A1	
$eu = -v_1 + v_2$	M1 A1	
$v_1 = \frac{u}{3}(1 - 2e)$; $v_2 = \frac{u}{3}(1 + e)$	M1 A1 A1	7

(b) $v_1 > 0 \Rightarrow \frac{u}{3}(1 - 2e) > 0 \Rightarrow e < \frac{1}{2}$ — M1 A1 c.s.o. — 2

(c)



$2mv_2 = 2mv_3 + 4mv_4$	M1	
$ev_2 = -v_3 + v_4$		
$v_3 = \frac{v_2}{3}(1 - 2e) = \frac{u}{9}(1 - 2e)(1 + e)$	M1 A1	
Further collision if $v_1 > v_3$		
i.e. if $\frac{u}{3}(1 - 2e) > \frac{u}{9}(1 - 2e)(1 + e)$	M1	
i.e. if $3 > 1 + e$ (as $(1 - 2e) > 0$)		
i.e. if $2 > e$	M1	
which is always true, so further collision occurs	A1 cso	6

[15]

11.	(a)	L.M. $2u = 2x + y$	M1 A1	
		NEL $y - x = \frac{1}{3}u$	M1 A1	
		Solving to $x = \frac{5}{9}u$ (*)	M1 A1	
		$y = \frac{8}{9}u$ (*)	A1	7
	(b)	$(\pm)\ \frac{8}{9}eu$	B1	
		L.M $\frac{10}{9}u - \frac{8}{9}eu = w$	M1 A1	
		NEL $w = \frac{1}{3}\left(\frac{5}{9}u + \frac{8}{9}eu\right)$	M1 A1	
		Solving to $e = \frac{25}{32}$	M1 A1	7
		accept 0.7812s		
	(c)	Q still has velocity and will *bounce back* from wall colliding with *stationary P*.	B1	1

[15]

1. There were also a significant number of fully correct answers to this question. Most candidates completed parts (a) and (b) well but part (c) proved to be rather more demanding.

For part (a) the majority of candidates understood and applied the conservation of linear momentum and the law of restitution correctly. The equations were usually consistent despite the occasional lack of a clear diagram, and only a very small minority got the restitution equation the wrong way round.

However, subsequent errors in the speed of *A* and *B* were common – these arose from simple sign errors in the initial equations or more commonly from minor processing errors. These basic errors can be costly – unexpected outcomes should be checked carefully to avoid continuing to work with unrealistic situations. A few candidates surprisingly failed to substitute ½ in for e and then struggled through with their answers for the rest of the question.

Full marks were usually scored for part (b) with only a minority of candidates with method errors in the use of the restitution equation. The question asked for the speed of B after the collision, so the final answer should have been positive, which was not always the case.

There were a wide variety of approaches to part (c). It was pleasing to see that some candidates could produce the given expression fluently with their methods clearly laid out. The most successful approach involved finding the separation of the two particles as *B* impacted with the wall and then to use the relative velocity to find the time taken to cover this separation. Some used the ratio of distances travelled or set up an equation in *T*.

Many could find the time to *B* hitting the wall and the distance travelled by *A* in this time, but got no further, having run out of time or having no idea how to proceed further.

A few solutions went off in entirely the wrong direction, either by thinking that another collision was needed (considering CLM and NEL again) or by attempting to use methods which implied non-zero acceleration. A worrying handful did not use the correct relationship between distance speed and time (e.g time = distance × speed was seen).

Expressions involving *a* and *u* were often badly written and these symbols would become interchanged during the rearrangement and simplification of terms. For example, a fractional term such as $\frac{5}{4}u$ was written without due care so that the *u* migrated to the bottom of the fraction during manipulation. Clear layout and a description of the symbols are vital in this kind of question to avoid these careless errors and to help examiners navigate through a candidates' work.

The given answer was helpful to some candidates who made a false start - realising their error they would often produce a better or correct solution. However, candidates working with incorrect values from part (a) were often misled into altering work displaying correct method in an attempt to derive the given answer. There were also some who tried to fudge incorrect processes to achieve the given answer.

2. Parts (a) and (b) were tackled with confidence by most candidates although a few were not sufficiently careful with signs and/or had long-winded algebraic manipulation to achieve the given result. CLM and the impact law were generally used correctly.

Part (c) proved to be more challenging and differentiated between the stronger candidates and those with confused concepts. No mention of *e* in the question caused some to ignore it or, more commonly, to assume the previous value. There were various arguments used to justify their final statement but, encouragingly, the range for *e* seemed to be understood.

3. Candidates made a confident start to this question, but in parts (a) and (b), poor algebraic skills and the lack of a clear diagram with the directions marked on it hampered weaker candidates' attempts to set up correct and consistent (or even physically possible) equations. The direction of *P* after impact was not given and those candidates who took its direction as reversed ran into problems when finding the value of *e*. Many realised that they had chosen the wrong direction and went on to answer part (b) correctly but some did not give an adequate explanation for a change of sign for their velocity of *P*. Algebraic and sign errors were common, and not helped by candidates' determination to reach the given answers.

Parts (c) and (d) caused the most problems. They could be answered using a wide variety of methods, some more formal than others. Many good solutions were seen but unclear reasoning and methods marred several attempts. Too many solutions were sloppy, with *u* or *d* appearing and disappearing through the working. A few words describing what was being calculated or expressed at each stage would have helped the clarity of solutions greatly. Students need to be reminded yet again that all necessary steps need to be shown when reaching a given answer. Too many simply stated the answer $\frac{3d}{5}$ without the explanation to support it.

4. Very few candidates noticed the link between part (a) of this questions and parts (b) and (c). This resulted in a considerable quantity of valid but unnecessary work. The marks allocated to the three parts of the question should give candidates an indication that each of the later parts is not expected to involve as much work as the first part.

(a) There were many substantially correct answers to this part. Most candidates formed correct equations using restitution and conservation of momentum. The difficulties started with the speed of P – many candidates whose answer was the negative of the printed answer did not justify the change of sign using the information about the value of e, and those whose answer agreed did not appreciate the need to verify that their value for velocity was in fact positive.
Candidates who changed the sign of their answer for the speed of P often went on to substitute incorrectly to find the speed of Q. There was also evidence of some confusion over the exact meaning of the question in part (aii), with several candidates starting by substituting □ in place of e.

(b) Most candidates elected to start the question afresh rather than use the results from part (a). Examiners were presented with confusing diagrams which were often contradicted by the working which followed. Alternatively there was no diagram and we had to decide for ourselves which direction the candidate assumed sphere *B* would move in after the collision with *C*.

(c) By this stage in the working many candidates were working with an incorrect initial speed of *B*, and were then further confused about the possible directions of motion of *A* and *B* after their collision. This often resulted in a page or more of working to deduce a velocity for *B* after the collision with *A*, all for a potential score of one mark. Most candidates did demonstrate an understanding that they needed to compare the speeds of *B* and *C* to determine whether or not there would be a further collision.

5. Many scored full marks in (a) and (b). It was pleasing to see that most used correct methods for momentum and impact law equations but disappointing to see a number of sign and arithmetic errors. Parts (c) and (d) proved to be more challenging. It did not help that some candidates confused themselves by giving every unknown speed the same name. Part (c) required the use of two correct equations solved simultaneously and many successfully showed that e = ¾. In (d) marks were often lost through poor explanation. For the final mark a clear statement backed up by a comparison of speeds was required.

6. In part (a) candidates generally understood the methods involved and were able to produce momentum and restitution equations. Inconsistencies between directions in diagrams and equations were common and many were unable to obtain a correct expression for the *speed* of *A* after the collision. In the second part, the rebound again caused problems with signs but most were able to set up the first inequality. The given double inequality was sometimes fudged or just stated without any attempt to justify $e > \frac{1}{4}$. In part (c), the correct method was usually adopted but accuracy errors were common.

7. The first part was very well answered with the odd error of incorrect signs. It was pleasing to note that only a few gave the restitution equation the wrong way round. Most of the errors here occurred in parts (b) and (c). There appeared to be some confusion over the fact that the KE of every particle before and after the impact was required and that these needed to be added and subtracted correctly in order to get a positive loss of KE. Some candidates missed out the KE of one of the particles and many subtracted to get a negative KE. In part (c) the impulse equation was often incorrect, with an incorrect sign in the velocity term. This led to a value of e >1. Some candidates realised that this was wrong and corrected their equation but others changed the final equation but failed to change the signs in their original impulse equation and were unable to score full marks.

8. The method in part (a) was generally well known and there were many good solutions. However, the second part proved to be more challenging with only the better candidates able to set up and solve their equations correctly and even then not all were able to pick up the final two marks - many only considered a further collision between B and C, which was impossible, instead of between A and B.

9. Part (a) was well done but parts (b) and (c) proved testing. Many produced irrelevant equations for a second collision between the spheres and wasted much time not realising that the question was whether or not a second collision occurred and not what the results of such a collision might be. The essential distinction between (b) and (c) is that in the former it is the direction of *P* after the first collision which is important whereas in (c) it is the magnitude and this was not recognised by many. It was not unusual to see a comparison of speeds in (b). In (c) there were many errors in manipulating the inequality and it was common to see the incorrect inequality $-3.6\,fu > -0.4u$ leading, through an incorrect sequence of steps, to the "correct" $f > \dfrac{1}{9}$.

10. (a) The method here was generally well known and the momentum equation was usually correct although the NIL equation sometimes had sign errors. Manipulation of the equations was generally poor.

(b) This was rarely correct – a common error was to compare the two velocities. Many answers were unsupported and got no credit.

(c) A significant number were able to write down attempts at the two equations but only a few were able to progress further as the algebra became more complex. A tiny number realised that the second impact was essentially the same as the first and were able to write down the final velocities without any further working. A full and convincing argument for this part was very rare.

11. The most notable characteristic about the majority of candidates' responses to this question was the number of completely correct solutions seen. The algebra in part (a) was quite straightforward but the same was definitely not true of part (b) and yet many could negotiate the minimum of four variables needed in this question and obtain the correct exact answer $e = \frac{25}{32}$. The principles needed to solve part (a) were well understood. When errors were made in part (b), these usually arose from not realising that there were two separate impacts to consider, one between *Q* and the wall, with an unknown *e* and a second impact between *P* and *Q*, in which $e = \frac{1}{3}$. Errors of sign were also seen, often resulting in a pair of incompatible equations for linear momentum and Newton's Experimental Law. In part (c), candidates were expected to explain that there would be a second impact between *Q* and the wall, which would result in a further impact between *Q* and the stationary *P*.